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Proof that there is no Simple Group whose Order lies between 1092 and 2001.

BY G. H. LING AND G. A. MILLER.

In this paper it is proposed to continue the search for simple groups of low orders. This work was begun by Hölder,* who examined all groups whose orders do not exceed 200. It was carried by Cole† up to the order 660 and by Burnside‡ up to the order 1092. It is here proven that no simple groups exist having orders between 1092 and 2001.

The work is, in general, based on the theorems of Sylow. For the sake of clearness it may be desirable to state the principal theorems used. In the first place, Burnside shows that all groups of odd order within the limits given are composite, and he proves also that a group of even order cannot be simple unless its order is divisible by 12, 16, or 56.§ We have also the theorems:

1. *The only simple groups, whose orders are the product of four or of five primes, are groups of order* 60, 168, 660 *and* 1092; *and no group whose order contains less than four prime factors is simple.*

2. *If H is one of n conjugate subgroups of a group G of order N, and if N is not a factor of $n!$, G cannot be simple.*||

3. *Groups of order* $p_1^\alpha p_2$, $p_1^\alpha p_2^2$, *and* $p_1^\beta p_2^\alpha$ $(\beta = 1, 2 \ldots . 5; p_1 < p_2)$ *are soluble.***

After rejecting all the orders not divisible by 12, 16, or 56, we have left 118 possible orders. We are enabled to reject the following ones:

By theorem 1: 1116, 1128, 1136, 1140, 1164, 1168, 1212, 1236, 1264,

* Math. Annalen, vol. XL, pp. 55-88.

† American Journal, vols. XIV and XV.

‡ Proceedings of London Mathematical Society, vol. XXVI (1895), pp. 333-339.

§ Burnside, "The Theory of Groups." Arts. 256, 260.

|| Hölder (l. c.).

** Frobenius, Berliner Sitzungsberichte, 1895, p. 190, and Burnside (l. c.).

1272, 1284, 1288, 1308, 1328, 1332, 1356, 1380, 1416, 1424, 1428, 1452, 1464, 1476, 1524, 1548, 1552, 1572, 1596, 1608, 1616, 1624, 1644, 1648, 1668, 1692, 1704, 1712, 1716, 1736, 1740, 1744, 1752, 1788, 1808, 1812, 1860, 1884, 1896, 1908, 1932, 1956, 1992.

By theorem 2: 1176, 1456, 1500, 1960.

By theorem 3: 1152, 1184, 1216, 1280, 1296, 1312, 1376, 1408, 1472, 1504, 1536, 1568, 1600, 1664, 1696, 1792, 1856, 1888, 1936, 1944, 1952, 1984, 2000.

By Sylow's theorem: 1248, 1360, 1368, 1392, 1488, 1632, 1760, 1776, 1840, 1904, 1968.

There remain 28 possible orders which require special treatment, viz.

$1104 = 2^4 . 3 . 23$,	$1120 = 2^5 . 5 . 7$,	$1188 = 2^2 . 3^3 . 11$,	$1200 = 2^4 . 3 . 5^2$,
$1224 = 2^3 . 3^2 . 17$,	$1232 = 2^4 . 7 . 11$,	$1260 = 2^2 . 3^2 . 5 . 7$,	$1320 = 2^3 . 3 . 5 . 11$,
$1344 = 2^6 . 3 . 7$,	$1400 = 2^3 . 5^2 . 7$,	$1404 = 2^2 . 3^3 . 13$,	$1440 = 2^5 . 3^2 . 5$,
$1512 = 2^3 . 3^3 . 7$,	$1520 = 2^4 . 5 . 19$,	$1560 = 2^3 . 3 . 5 . 13$,	$1584 = 2^4 . 3^2 . 11$,
$1620 = 2^2 . 3^4 . 5$,	$1656 = 2^3 . 3^2 . 23$,	$1680 = 2^4 . 3 . 5 . 7$,	$1728 = 2^6 . 3^3$,
$1764 = 2^2 . 3^2 . 7^2$,	$1800 = 2^3 . 3^2 . 5^2$,	$1824 = 2^5 . 3 . 19$,	$1836 = 2^2 . 3^3 . 17$,
$1848 = 2^3 . 3 . 7 . 11$,	$1872 = 2^4 . 3^2 . 13$,	$1920 = 2^7 . 3 . 5$,	$1980 = 2^2 . 3^2 . 5 . 11$.

We now proceed to the consideration of these cases.

$$1104 = 2^4 . 3 . 23.$$

A group of order 1104 must contain 1 or 24 conjugate subgroups of order 23. If it were simple, it would, therefore, be represented as a doubly-transitive group of degree 24. The maximal group of degree 23 would be of order 46. As this group of order 46 would contain negative substitutions, a group of order 1104 cannot be simple.

$$1120 = 2^5 . 5 . 7.$$

A group of order 1120 must contain 1 or 8 subgroups of order 7. A simple group of this order could, therefore, be represented as a doubly-transitive group of degree 8. Since there is no transitive group of degree 7 and order 140,* this is impossible.

$$1188 = 2^2 . 3^3 . 11.$$

* Mathieu, Comptes Rendus, vol. XLVI, p. 1048.

If simple, this group has 12 conjugate subgroups of order 11. It could then be represented as doubly-transitive in 12 elements. But there is no such group of this order.*

$$1200 = 2^4 . 3 . 5^2.$$

A simple group of order 1200 could be represented as a transitive group of degree 16, since every group of order 1200 must contain 1, 6 or 16 subgroups of order 25. This group could not be imprimitive, since there is no group of degree 8 and order 1200.† It could not be primitive since there is no primitive group of degree 16 and order 1200.‡

$$1224 = 2^3 . 3^2 . 17.$$

Since every group of this order contains 1 or 18 subgroups of order 17, a simple group of this order could be represented as a doubly-transitive group of degree 18. The maximal subgroup of degree 17 would be of order 68. The group of order 1224 would, therefore, contain a subgroup of order 8 that contains a substitution formed by four cycles of order 4 and substitutions of degree 18. Since the latter would have to be negative, the simple group is impossible. It might appear that the group of order 8 could contain 5 substitutions of order 2 and degree 16, but this is impossible, since there would be just 17.9 conjugate substitutions of this kind.

$$1232 = 2^4 . 7 . 11.$$

Every group of order 1232 must contain 1 or 56 conjugate subgroups of order 11. A simple group of order 1232 would then contain 560 operators of order 11. Hence, it could not contain 176 conjugate subgroups of order 7. It would then have to contain 22 conjugate subgroups of order 7, and could be represented as a transitive group of degree 22. There would be subgroups of order 56 and degree 21. Such a subgroup must contain 28 operators which transform the operator of order 7 into itself. This group would contain operators of order 14 and degree 21. But these operators would be negative. Hence, no simple group of this order exists.

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* Miller, Quar. Jour. Math., vol. XXVIII, pp. 193–231.

† Mathieu (l. c.), p. 1208.

‡ Miller, Amer. Jour. Math., vol. XX, p. 229.

$$1260 = 2^2 . 3^2 . 5 . 7.$$

A group of order 1260 has 1, 15, or 36 conjugate subgroups of order 7. If there are 36 such groups, each of them is self-conjugate in a subgroup of order 35, which is necessarily cyclical and whose operators are circular in 35 elements if the group is represented as a transitive group of degree 36. All the operators then affect 35 or 36 elements. There is no group of this order and type.*

The group, if simple, would, therefore, have 15 conjugate subgroups of order 7, and could be represented as a primitive group of degree 15. But there is no such simple group.†

$$1320 = 2^3 . 3 . 5 . 11.$$

A simple group of this order would have 12 conjugate subgroups of order 11 and could be represented as a doubly-transitive group of degree 12. There is only one doubly-transitive group of this order and degree, and it is composite.‡

$$1344 = 2^6 . 3 . 7.$$

A simple group of this order would have 21 conjugate subgroups of order 64. The groups of order 64 cannot be distinct. If the groups are not distinct we get, by means of the equation,§

$$1 + 2x_1 + 4x_2 + 8x_3 + 16x_4 = 21,$$

either (a) $x_1 \neq 0$ or (b) $x_1 = 0$, $x_2 \neq 0$.

(a). Let $x_1 \neq 0$. There are then several subgroups of order 64 which have in common a subgroup of order 32. The latter is then self-conjugate in a group of order $64(1 + 2k)$ which has 3 or 7 conjugates. The main group could then be represented as transitive in 7 elements, in which case it could not be simple.||

(b). Let $x_1 = 0$ and $x_2 \neq 0$. Then several groups of order 64 have in common a subgroup of order 16 which is then self-conjugate in a subgroup of order $32(1 + 2k)$. The latter group then has $\dfrac{64 . 21}{32(1 + 2k)}$ conjugates. The only case that need be dealt with is that in which $k = 1$. The main group can then

* Jordan, Liouville's Journal, 2ième ser., vol. XVII, pp. 351–367.

† Miller, Proc. London Math. Soc., vol. XXVIII, pp. 533–545.

‡ Miller, Quar. Journal, vol. XXVIII, p. 193.

§ Burnside, Proc. London Math. Soc., vol. XXVI, p. 336.

|| Mathieu (l. c.), p. 1208.

be represented as transitive in 14 elements. There are three transitive groups of order 1344 and degree 14, but all are composite.* There is then no simple group of this order.

$$1400 = 2^3 . 5^2 . 7.$$

A simple group of order 1400 would have 56 conjugate subgroups of order 25 and 8 or 50 conjugate subgroups of order 7. If the subgroups of order 25 should be all distinct, it is clear that there could be at most 8 operators of even order and, if the group exists, a self-conjugate subgroup of order 8.

If the subgroups of order 25 should not be distinct, there would be a subgroup of order 5 self-conjugate in the whole group. There is then no simple group of this order.

$$1404 = 2^2 . 3^3 . 13.$$

A simple group of this order would have either 13 or 52 conjugate subgroups of order 27. The group could not be represented as a transitive group of degree 13.† Hence, there must be 52 conjugate subgroups of order 27, and the group could be represented as primitive in 52 elements. Hence, it would have a subgroup of order 27 and degree 51. This group would have at least one transitive constituent of degree 3. There must then be a subgroup of order 9 which would be common to several of the groups of order 27, and hence self-conjugate in the group generated by them. This would require that the groups of order 27 should not be maximal. Consequently there can be no simple group of order 1404.

$$1440 = 2^5 . 3^2 . 5.$$

If a group of order 1440 were simple, it would contain either 96 or 36 conjugate subgroups of order 5.

If it would contain 96 such subgroups, it could be expressed as a transitive group of degree 96. If it were primitive, then it would have a subgroup of degree 95 and order 15, which would be cyclical. Consequently all the transitive constituents of this latter group would be of degree 15. This is impossible, since $95 \div 15 \neq$ integer. If the group were imprimitive in 96 elements, it could be primitive and of degree either 48, 32 or 24. A primitive group of order 1440, and of degree 48, would have 48 subgroups of order 30 and degree

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*Miller, Quar. Jour., vol. XXIX, p. 248.

†Ibid., p. 224.

47, each of which would contain *one* subgroup of order 5. We would not then have 96 subgroups of order 5. A primitive group of order 1440 and degree 32 would have 32 subgroups of order 45 and degree 31, each of which would contain *one* subgroup of order 5, giving in all 32 subgroups of order 5. A primitive group of order 1440 and degree 24 would have 24 subgroups of order 60 and degree 23. These could contain at most 6 subgroups of order 5 and degree not greater than 20. Hence, the total number would be at most $\frac{6 \cdot 24}{4} = 36$ There cannot then be 96 subgroups of order 5.

If the group has 36 conjugate subgroups of order 5, it can be represented as a primitive group of degree either 18 or 36. In the first case the group would contain 18 subgroups of order 80 and degree 17. These must contain 16 subgroups of order 5, and, consequently, a self-conjugate subgroup of order 16 and degree <17. These subgroups of order 80 could not then be maximal. The group then must be primitive in 36 elements. There must be 36 subgroups of order 40 and degree 35, each containing a substitution of degree 35 and order 5, and at least one of its transitive constituents must be of degree 5. The order of this constituent could not be 5, for the order of each constituent must be divisible by the same prime numbers. If the order were 20, the group of order 40 and degree 35 would contain a substitution of order 2 and degree 20 that would be commutative to each one of its substitutions. Hence, this group of order 40 and degree 35 would contain at least $35 - 20 = 15$ substitutions of degree 20 and order 2. This is impossible, since only the 10 which correspond to substitutions of order 2 in the constituent group of degree 5 and order 20 could be of order 2.

It remains to consider the case in which the order of the constituent of degree 5 is 10. The group of order 4 which corresponds to identity of this constituent has at least one substitution of order 2 and degree 20 which is commutative to every substitution of the group of order 40 and degree 35. Hence, the latter group must contain at least 15 other substitutions of degree 20 and order 2. We shall divide the problem into three parts: (1) when the subgroup of order 4 is cyclical, (2) when it is non-cyclical and of degree 20, (3) when it is non-cyclical and of degree 30.

(1). In this case the substitutions of order 4 must contain 5 cycles of order 4 and 5 of order 2, and there must be a transitive constituent of degree 20 and order 40 that contains a cyclical subgroup of order and degree 20. The

operators in the tail of this group must transform the operators of the head either into the 9^{th} or into the 19^{th} power. In the former case there would be only 10 operators of order 2 in the tail, and in the latter case the degree of each of the operators of order 2 in the tail of the group of order 40 and degree 35 would exceed 20, since the degree of the part of these operators that belongs to the transitive constituent of degree 20 could not be less than 18.

(2). In the second case the given group of order 4 and of degree 20 is found in 15 conjugates of the group of order 40 and degree 35. All the subgroups which correspond to this group of order 4 and degree 20 in these conjugates must be contained in the first group of order 40, and a substitution that belongs to the group of order 4 cannot be transformed into itself by every substitution of a conjugate to the first group of order 40. Hence, it is impossible to construct the required 15 subgroups of order 4 with the substitutions of the first group of order 40.

In the third case, any two of the substitutions of order 2 in the group of degree 30 and order 4 have 10 elements in common. If each of these substitutions of order 2 is commutative to every substitution of the subgroup of order 40, then every one of the conjugates of this group of order 40 must contain substitutions of the group of order 40 for its 4 operators that are commutative to every operator of the group. This is impossible, since no two of these groups of order 4 can contain any common operator except identity. If only one of the three operators of order 2 in the given subgroup of order 4 is commutative to all the substitutions of the group of order 40 and degree 35, this group must contain a transitive constituent of degree 20 and order 40 whose tail contains only substitutions of degree 20. Hence, this group of order 40 and degree 35 cannot contain 15 substitutions of degree 20 and order 2. This completes the proof that there is no simple group of order 1440.

$$1512 = 2^3 . 3^3 . 7 .$$

In a simple group of this order there would be 36 subgroups of order 7. The group could, then, be represented as a transitive group of degree 36. It could not be represented as an imprimitive group of degree 36, for it could then be represented as a primitive group of degree 18, and hence would contain a group of order 84 and of degree 17 which would have only one subgroup of order 7. In this case there would not be 36 subgroups of order 7. The groups of order

1512 and degree 36 must then contain a maximal subgroup of order 42 and of degree 35. Since the order of each transitive constituent of this subgroup must involve tho same prime factors,* this group would consist of a simple isomorphism between groups of order 42. If one of these constituents should be of degree 7, the group of order 42 would be simply isomorphic to the group of order 42 and of degree 7. If none of the constituents should be of degree 7, the constituents would be of degrees 14 and 21. There are only two transitive groups of degree 14 and of order 42. As one of these contains only a single substitution of order 2, it cannot be represented as a transitive group of degree 21. The group of order 42 would then have to be simply isomorphic to the metacyclic group of degree 7. This latter group can be represented as a transitive group of each of the degrees 7, 14, 21 and 42. In each case it contains negative substitutions. Hence, the group of order 42 and of degree 35 would have to contain an even number of transitive constituents, viz. 14, 7, 7, 7 or 14, 21. In the former case, the main group would be of class 30, and in the latter, of class 32. In the former case the substitutions of order 2 and 6 would be of degree 32; in the latter case, those of order 3 would be of degree 33.

If the group of order 42 and of degree 35 should have the constituents 14, 7, 7, 7, it would contain $\frac{7 \cdot 36}{4} = 63$ subgroups of order 6, each of which would be generated by a substitution containing 5 cycles of order 6 and one transposition. The group of order 24 which would transform such a subgroup into itself, would therefore contain a transitive constituent of degree 2. As the group of order 42 could not contain a subgroup of order 12, this is impossible.

It remains to consider the case in which the group of order 42 and of degree 35 would involve the two constituents of degrees 14 and 21. The group of order 24 which would transform one of its substitutions of order 2 into itself would have to contain 4 cyclical subgroups of order 6 and of degree 35. Hence, the subgroup of order 8 contained in the whole group would have to be transformed into itself by at least 24 substitutions. If this subgroup of order 8 should be transformed into itself by just 24 substitutions, there would be 63 such subgroups, and the main group could be represented as a transitive group of degree 63. This could not be primitive since the group of order 8 in the group of degree 62 and of order 24 would have to contain a constituent of degree 2.

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* Jordan, "Traité des Substitutions," p. 284.

If it were imprimitive, it would be simply isomorphic to a primitive group of degree 21. The group of order 72 and degree 20 would then contain 3 subgroups of order 8 and of degree 20. These would be transformed according to the symmetric group of degree 3, and hence the group of order 72 would have to contain a self-conjugate subgroup of order 4 and of degree 20. The group of order 72 would contain 4 subgroups of order 9, which it would transform according to the alternating or symmetric group of degree 4, and hence it would contain also a self-conjugate subgroup of order 3 and of degree 12. It would, therefore, have to contain a constituent of degree 12, which would be either transitive or would contain two constituents of degree 6. The substitutions of order 3 in the constituent of degree 8 would have to be of degree 6. As the constituent of degree 12 could not contain $20 - 12 = 8$ substitutions of degree 6 and order 3, the whole group could not contain 63 subgroups of order 8.

If the groups should contain 21 subgroups of order 8, the degree of each of the transitive constituents contained in the self-conjugate subgroup of order 8 which occurs in the subgroup of order 72 and degree 20, would be divisible by 4. It can be proven as before that this group of order 72 would contain a self-conjugate subgroup of order 3 and degree 12. The constituent of degree 12 would be transitive and the proof given above would apply to this case.

It is impossible that the number of subgroups of order 8 should be 27 or 189, since the number of substitutions that transform the subgroup of order 8 into itself is divisible by 3.

$$1520 = 2^4 . 5 . 19.$$

A simple group of this order would have 20 conjugate subgroups of order 19. The group could be represented as doubly-transitive and of degree 20. The group of order 76 and degree 19 would have a substitution of order 38 which could not be represented by means of 19 elements. Consequently there can be no simple group of this order.

$$1560 = 2^3 . 3 . 5 . 13.$$

A simple group of this order would have 40 conjugate subgroups of order 13, and 26 or 156 conjugate subgroups of order 5. The group could then be represented as transitive and of degree 40. If the subgroups of order and degree

39 were transitive, the operators of the group would all be of degree 39 or 40. No such group of this order can exist.* If the subgroups of order 39 were intransitive, each of them would contain 26 operators of degree 36, forming 13 conjugate subgroups of order 3 and degree 36. The whole group would then contain $\frac{40 \cdot 13}{4} = 130$ conjugate subgroups of order 3. The subgroups of order 3 are self-conjugate in groups of order 12 which would contain operators of order 6. From the consideration of the representation of the group as transitive and of degree 130, it becomes clear that there would be 129 conjugate subgroups of order 6. Since the groups of order 5 are not commutative to the groups of order 3, it follows that the number of groups of order 5 must be 126. The group would then contain 480 operators of order 13, 624 of order 5, and 520 of order 6 or 3. But this is impossible. Hence there is no simple group of this order.

$$1584 = 2^4 \cdot 3^2 \cdot 11.$$

A simple group of this order would have 12 or 144 conjugate subgroups of order 11. The group cannot be transitive and of degree 12. Hence, it must be transitive and of degree 144. In this case all of the operations would affect 144 or 143 elements. Such a group cannot exist.†

$$1620 = 2^2 \cdot 3^4 \cdot 5.$$

A simple group of this order would have 10 conjugate subgroups of order 81. But there is no transitive group of order 1620 and degree 10.‡ Hence, there is no simple group of this order.

$$1656 = 2^3 \cdot 3^2 \cdot 23.$$

There would be 24 conjugate subgroups of order 23 in a simple group of this order. The group could then be represented as doubly-transitive and of degree 24. The subgroup of order 69 and degree 23 would be cyclical. But no such subgroup exists. There is then no simple group of this order.

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*Jordan, Liouville's Jour., 2ième ser., vol. XVII, pp. 351–367.

† Jordan (l. c,).

‡ Cole, Quar. Jour., vol. XXVII, p. 39.

$$1680 = 2^4 . 3 . 5 . 7.$$

The number of subgroups of order 5 in a simple group of order 1680 would be 16, 21, 56, or 336. There would also be 120 subgroups of order 7, so that there could not be 336 of order 5. There could not be 16 subgroups of order 5, for there is no simple group of this order, primitive and of degree 8 or 16. The number of groups could not be 56, for the group would then permute them primitively or imprimitively according to a group of degree 56. If primitively, there would be a subgroup of order 30 and of degree 55 which would contain a cyclical group of order 15 and whose systems of intransitivity would consequently be multiples of 15. But $55 \div 15 \neq$ integer. Hence, this case could not occur. If imprimitively, the only case that requires consideration is that in which the systems of imprimitivity contain 2 elements. There would be operations leaving some system unchanged, and such an operation, if it left one of the elements unchanged, would have to leave the other unchanged. But a subgroup of order 5 leaves itself unchanged and does not leave unchanged any other group of order 5. The number of subgroups of order 5 could not, therefore, be 56. If the number were 21, there would be a subgroup of order 80 and of degree 20 in which the group of order 5 would be contained self-conjugately. In this group 20 operators would be commutative to the operations of order 5, and would form a regular group. The group of order 80 and degree 20 would then be transitive, and the average degree of this operation would be 19. The average degree of the subgroup of order 20 is 19. Hence, the average degree of the remaining 60 operators would have to be 19. If each were of degree 19, this would give rise to 630 distinct operations. In addition, there would be 399 from the groups of order 20 and 720 from the groups of order 7, or in all more than 1680. It can be shown that, in case all of the tail of 60 in the group of order 80 were not of degree 19, the total number would be still higher Hence, the group could not have 21 subgroups of order 5 and no simple group of the order could exist.

$$1728 = 2^6 . 3^3.$$

A group of this order has 1, 9, or 27 conjugate subgroups of order 64. The group, if simple, could not be transitive and of degree 9.* Hence, it could be rep-

* Cole, Quar. Jour., vol. XXVI, p. 386.

resented as primitive and of degree 27. It has then a subgroup of order 64 and degree 26, one of whose transitive constituents is of degree 2. Hence, there is a group of order 32 common to several subgroups of order 64 which are then not maximal. Hence, there is no simple group of this order.

$$1764 = 2^2 . 3^2 . 7^2.$$

The order of this group is of the form $p^2 q^2 r^2$. Consequently the group is composite.*

$$1800 = 2^3 . 3^2 . 5^2.$$

A simple group of this order would contain 36 subgroups of order 25 and could, therefore, be represented as a transitive group of degree 36. The subgroup of degree 35 and order 50 could not be maximal because its subgroup of order 25 would contain a transitive constituent of degree 5. Hence, the group, if simple, must be primitive in 18 elements. In this case the subgroup of degree 17 would be of order 100, and hence the group could not contain 36 subgroups of order 25.

$$1824 = 2^5 . 3 . 19.$$

A group of this order has 1 or 96 conjugate subgroups of order 19. If it had 96 such subgroups, it could be represented as transitive and of degree 96. It would have subgroups of degree 95 and order 19, all of whose operations would affect 95 elements. All of the operations of the group would then affect 96 or 95 elements. Such a group cannot exist.† Hence, a group of this order must be composite.

$$1836 = 2^2 . 3^3 . 17.$$

A simple group of order 1836 would have 18 conjugate subgroups of order 17, and hence could be represented as doubly-transitive and of degree 18. There would then be a subgroup of degree 17 and order 102. But no such group exists.‡ Hence, there is no simple group of this order.

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*Maillet, Quar. Jour., vol. XXIX, p. 250.

†Jordan (l. c.).

‡Miller, Quar. Jour., vol. XXXI, pp. 49–57.

$$1848 = 2^3 . 3 . 7 . 11.$$

A simple group of order 1848 would have 12 or 56 conjugate subgroups of order 11, and 22 conjugate subgroups of order 7. There is no group of this order which is transitive and of degree 12. There must then be 56 conjugate subgroups of order 11, each self-conjugate in a subgroup of order 33. Representing the group as transitive and of degree 22, the operator of order 11 would have 2 cycles and the group of order 33 could not be constructed. Then no simple group of this order can exist.

$$1872 = 2^4 . 3^2 . 13.$$

Any group of this order has 1 or 144 conjugate subgroups of order 13. If there were 144 such subgroups, the group could be represented as transitive and of degree 144. All of the operations of the group would then affect either 143 or 144 elements. Such a group of this order cannot exist.* Hence, all groups of this order are composite.

$$1920 = 2^7 . 3 . 5.$$

A simple group of this order would have 15 conjugate subgroups of order 128. But there is no simple primitive group of this order and of degree 15.† Hence, there is no simple group of this order.

$$1980 = 2^2 . 3^2 . 5 . 11.$$

A group of this order, if simple, would have 12 or 45 conjugate subgroups of order 11. There is no doubly-transitive group of order 1980 and degree 12. Hence, there would be 45 conjugate subgroups of order 11. The group could be represented as transitive and of degree 45. The group of order and degree 44, which leaves one element unaffected, could not be transitive, for then all of the operators of the whole group would be of degree 44 or 45. Such a group of this order cannot exist.‡ If the group of order and degree 44 is intransitive, the operations of order 11 are transformed into themselves by 22 substitutions. Hence, the group contains a cyclical subgroup of order 22 and degree 44. The

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* Jordan (l. c.). † Miller, Proc. London Math. Soc., vol. XXVIII. ‡ Jordan (l. c.).

tail of this group contains 11 operations of order 2 and degree 44, and 11 operations of order 2 and degree 40. There are then $\frac{11\times 45}{5} = 99$ conjugate groups of order 2 and degree 40. Representing the group as transitive and of degree 99, the subgroup leaving one element unaffected is of order 20, and contains *one* subgroup of order 5 which leaves unaffected $4 + 5k$ elements, and consequently has $\frac{99}{4 + 5k}$ conjugates. It is easy to show, however, that the number of subgroups of order 5 must be either 36 or 66. Hence, there can be no simple group of order 1980.

ITHACA, *July*, 1899.